Resource Listing for Differential Equation With Emphasis on Modeling and Technology

(Last updated by Brian Winkel, Director SIMIODE, on 16 September 2015)

Most of the materials in this list are free through some sponsored or open source effort or committed individuals to a free and open exchange and have some modeling content in addition to thorough text narrative on theory and techniques. Most often they are readily available for download, usually in pdf version, but sometimes in other versions as well, e.g., TeX. We comment on them below after citation.

The reader can click on the Table of Contents Entry to get to these various materials and once there can click on the Entry title to get back to this Top of the Document page. The file is completely searchable.

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Text Books

(All of these materials are FREE and appear in author alphabetical order.)

(1) Conrad, Bruce. P. 2010. Ordinary Differential Equations: A Systems Approach.
https://math.temple.edu/~conrad/ODEgreen.pdf and
https://math.temple.edu/~conrad/ODEblue.pdf

This is an author posted second edition to the text which was originally published by Prentice Hall in 2003.

The pdf is heavily peppered with very helpful hot-links to definitions, equations, statements and solutions to exercises which can be hot-linked back and forth, and other issues in the text for reference, solutions are provided. The book has everything, motivation by models, lots of exercises and solutions, theory with readable proofs. As an example in the first chapter on First-Order Differential Equations there are models, lots of attempts to reach out and touch other fields, e.g., biology and chemistry, solution strategies, e.g., separation of variables and integrating factors, numerical methods, geometric analysis, and more.

The author treats second order linear differential equations in conjunction with linear systems and also discusses nonlinear systems. Much of the material is treated through geometric approaches with emphases on such notions as phase plane, stability, equilibrium, etc.

The author says, "The text is available from either of the links below. Within the text you will find hyperlinks, which are in red. These will take you to one of the following:

- A numbered equation or theorem
- A figure
- The answer to an odd-numbered exercise
- The definition of a word.

"Hyperlinks in the answers to the exercises will are available to return to the exercise or to get a detailed answer. If you follow a hyperlink to a place other than an exercise answer, you will need to get back to the place you were reading.

"The purpose of linking certain words to their definitions is to remind you that it is important to know the definitions of these words, and to give you the opportunity to view the definitions as needed. To help you quickly locate the relevant definitions, I have used an alternate color for the word being defined. One version of the text uses green; the other uses blue. There are no other differences in the versions of the text. Please select the version that works best on your computer's screen."

There is an errata file https://math.temple.edu/~conrad/ODE_errata/Errata0.pdf which the author keeps up to date.

(2) Ekol, Georg L. 2010.l Differential Equations. Accessed 15 February 2015.
http://oer.avu.org/ .

From the Open Educational Resources by the African Virtual University, this text (in .doc and .pdf version) with introduction and overview video and is meant to be used as a classroom guide as well as a text with small group work. There are no applications offered. From the web page we quote,

"This module consists of two units, namely Introduction to ordinary differential equations and higher order differential equations respectively. In unit one both homogeneous and non-homogeneous ordinary differential equations are discussed and their solutions obtained with a variety of techniques. Some of these techniques include the variation of parameters, the method of undetermined coefficients and the inverse operators. In unit two series solutions of differential equations are discussed. Also discussed are partial differential equations and their solution by separation of variables. Other topics discussed are Laplace transforms, Fourier series, Fourier transforms and their applications."

(3) Finan, Marcel B. 2015. Electronic Textbooks. Accessed 8 March 2015.
http://faculty.atu.edu/mfinan/nnotes.html

Here you will find a whole collection of differential equations texts:

- A First Course in Elementary Differential Equations;
- A First Course in Elementary Differential Equations: Problems and Solutions;
- Laplace Transforms: Theory, Problems, and Solutions;
- A Second Course in Elementary Differential Equations;
- A Second Course in Elementary Differential Equations: Problems and Solutions;
- A First Course in Quasi-Linear Partial Differential Equations for Physical Sciences and Engineering; and
- A First Course in Quasi-Linear Partial Differential Equations for Physical Sciences and Engineering: Solution Manual.

The texts combine theory with proofs and examples of techniques with very little modeling. The proofs are very readable. So if students wish to delve into theory this is a good place to read.

(4) Joyner, David. 2007. Introductory Differential Equations Using SAGE. Accessed 15 May 2014. http://sage.math.washington.edu/home/wdj/teaching/DiffyQ/des-book.pdf .

This pdf text offers rather direct (not much modeling) coverage of the basic differential equations through boundary value, Fourier series, and partial differential equations (heat and wave) with SAGE code at every opportunity to show how the grinding mathematics can be easily done in SAGE.

(5) Lebl, Jiri. 2014. Notes on Diffy Qs: Differential Equations for Engineers. Accessed 1 May 2014. http://www.jirka.org/diffyqs/

There is a pdf and html version of the text as well as a purchasable paperback version. The text has pretty much all the basics, but has only a few exercises and there is very little modeling. However, the sections on second order differential equations do consider frequency response issues. The author claims, "The book has been selected as an Approved Textbook in the American Institute of Mathematics Open Textbook Initiative."

The author says,

"A one semester first course on differential equations, aimed at engineering students. Prerequisite for the course is the basic calculus sequence. This free online book (e-book in webspeak) should be usable as a stand-alone textbook or as a companion to a course using another book such as Edwards and Penney, *Differential Equations and Boundary Value Problems: Computing and Modeling* or Boyce and DiPrima, *Elementary Differential Equations and Boundary Value Problems* (section correspondence to these two is given). I developed and used these notes to teach Math 286/285 at the University of Illinois at Urbana-Champaign (one is a 4-day-a-week, the other a 3-day-a-week semester-long course)."

Several members of the SIMIODE community (John Thoo and Dina Yagodich) have used this text and found the material to be quite good. Moreover, the author is quite responsive to correspondence.

(6) Sloughter, Dan. 2015. Difference Equations to Differential Equations. Accessed 8 March 2015. http://www.synechism.org/wp/difference-equations-to-differential-equations/ **.**

This is a calculus book with the last chapter devoted to ordinary differential equations topics. The effort begins with a study of numerical methods and goes through traditional technique strategies such as first order and second order linear differential equations, with sections on series solutions and phase plane approaches. There are very few applications in the text to motivate the differential equations under study. However, there is a section on "pendulums and mass-spring systems." At the end of each section there are technique problems and some modeling problems.

(7) Terrell, Robert. 2014. *Notes on Differential Equations* (Version 5: 2014) . Accessed 8 March 2015.

http://www.math.cornell.edu/~bterrell/dn.pdf

The author offers Octave code for this text. In the Preface the author says, "I may point out that the title is not Solving Differential Equations; we derive them, discuss them, review calculus background for them, apply them, sketch and compute them, and also solve them and interpret the solutions. This breadth is new to many students." To which we might add, "and to faculty."

There is a nice feature the author describes at the start of the text, "Some of the exercises have the format, `What's rong with this?' These are either questions asked by students or errors taken from test papers of students in this class, so it could be quite beneficial to study them."

There is a very readable mix of theory, application, computer algebra system which is very refreshing.

The material does discuss the usual suspects, heat and wave partial differential equations as well as the vibration of a drum head, but does cover Laplace Transform. The path to Fourier series is historical and very interesting. The last chapter is a "tribute" to chaos with attempts to model weather, well, futile attempts!

(8) Trench, William F. 2013. *Elementary Differential Equation*. Books and Monographs. Book 8. Accessed 1 May 2014. http://digitalcommons.trinity.edu/mono/8 .
Trench, William F. 2013. *Elementary Differential Equations with Boundary Value Problems*. Books and Monographs. Book 9. Accessed 1 May 2014. http://digitalcommons.trinity.edu/mono/9 .

Both texts are also available at Scholar Commons of the University of South Florida Textbook Collections. http://scholarcommons.usf.edu/oa_textbooks/9/ .

Both texts also available as Approved Textbooks at the American Institute of Mathematics (AIM) at http://aimath.org/textbooks/approved-textbooks/ .

This is the most widely cited and referenced free online text for differential equations and is very complete.

Annotation: These two books are exactly what they say they are. Moreover, they are FREE. Go to the Digital Commons site at Trinity University, San Antonio TX USA (http://digitalcommons.trinity.edu/) and you can freely download either version and a number of other texts. The author has gone to great lengths to offer a reasonable level of theory for first course, a great number of worked examples and illustrations of techniques and applications, and a good number of interesting applications problems grouped at the end of reasonable sections occur in the first two general areas of study for ordinary differential equations, e.g., first order and second order ordinary differential equations. While there are scattered about a few application type exercises throughout the text beyond these first two set of applications, the text is mostly a rich treatise and an informal style conversation with the reader about the techniques and usefulness of the various solution strategies of ordinary (and in the case of Book 9 partial differential equations). These texts have had years of use through commercial publishers and the author has now made them freely available to the public in this Open Access venue. It is interesting to note that these books are very high in the Top 10 Faculty Downloads but are not in the Top 10 Student Downloads. Nevertheless, we found the texts to be very comprehensive, very readable, engaging, and mathematically correct and complete. We believe a teacher could send a student to a specific section, ask them to read the material, follow the examples, and submit some of the exercises as evidence of command of the technique, perhaps to be followed up by an exam or quiz question.

In the Preface to Book 9 the author says

- An elementary text should be written so the student can read it with comprehension without too much pain. I have tried to put myself in the student's place, and have chosen to err on the side of too much detail rather than not enough.

• An elementary text can't be better than its exercises. This text includes 2041 numbered exercises, many with several parts. They range in difficulty from routine to very challenging.
• An elementary text should be written in an informal but mathematically accurate way, illustrated by appropriate graphics. I have tried to formulate mathematical concepts succinctly in language that students can understand. I have minimized the number of explicitly stated theorems and definitions, preferring to deal with concepts in a more conversational way, copiously illustrated by 299 completely worked out examples. Where appropriate, concepts and results are depicted in 188 figures.

Although I believe that the computer is an immensely valuable tool for learning, doing, and writing mathematics, the selection and treatment of topics in this text reflects my pedagogical orientation along traditional lines.

All the formulae one could want are offered, and in most cases, derived in these texts. In the sections on Numerical methods there are rich illustrations with tables and comparisons of methods to motivate the value and need for numerical approaches. The Heat Equation is just presented with no derivation or intuitive motivation while the Wave Equation is thoroughly presented and motivated.

Further, with respect to technology, in the section on Fourier Series of Book 9, p. 599 the author says,

USING TECHNOLOGY
"The computation of Fourier coefficients will be tedious in many of the exercises in this chapter and the next. To learn the technique, we recommend that you do some exercises in each section `by hand,' perhaps using the table of integrals at the front of the book. However, we encourage you to use your favorite symbolic computation software in the more difficult problems."

(9) **Waterman, Gregg. 2014. Ordinary Differential Equations for Ordinary People. Accessed 15 September 2015**.
http://math.oit.edu/~watermang/math_321/321_book_4-19-14.pdf

This pdf text is very nicely written with "learning outcomes" and "performance criteria" in each section, chapter summaries, and many motivational examples and illustrations for science and engineering students. There are section by section worked examples and applications, exercises and occasional "Theoretical concerns" subsections in the text which are good to think about. The author uses words that engineering students should be comfortable.

The Table of Contents is hot-linked to permit the reader to jump right to the desired section instead of scrolling. There is also a hot-linked index at the back of the book which enables readers to get immediately to what they are looking for. Even though there are not any deep physical derivations there are plausibility discussions and reality checks upon completion of a topic, e.g., the beam equation.

The author says, "This book is designed for engineering students rather than mathematics students, with an emphasis on the roles of parameters and initial/boundary conditions, and interpreting ODEs and their solutions physically whenever possible."

This text deals only with ordinary differential equations topics and has no partial differential equations sections. However, there is a thorough section on boundary value problems.

The author has a web page for his differential equations course at Oregon Institute of Technology, http://math.oit.edu/~watermang/math_321/321_index.html . Here, in addition to the complete text divided out by reasoned chapters, he lists many resource, MANY(!), for his students and anyone who visits his rich site.

BTW in addition to his differential equations course the author has written a number of other on-line texts: Intermediate Algebra, College Algebra, Introductory Linear Algebra, Mathematical Foundations, and Introductory Mathematical Probability.

The author has a YouTube channel at https://www.youtube.com/channel/UC2G5uTfYsTHhPGuX_JCj9Uw in which there are many worked examples and lessons. We list them here by name and number: Laplace Transform (16), Linear Algebra (13), Ordinary Differential Equations (12), and Systems of Differential Equations (4).

(10)**Hirsch, M. W., S. Smale, and R. L. Devaney, 2004.** ***Differential Equations, Dynamical Systems, and Intro to Chaos, Second Edition*****. Elsevier**. http://www.math.upatras.gr/~bountis/files/def-eq.pdf / Accessed 14 May 2015. This classic text is freely available at this site.

(11)**Markovich, Peter A. 2006. *Partial Differential Equations: A Visual Approach*. Springer Monograph.** http://homepage.univie.ac.at/peter.markowich/galleries/vortrag.pdf . Accessed 15 May 2015. In 99 frames with lots of beautiful images and no mathematics the writer offers the following set of topics:

- Gas dynamics Boltzmann equation
- Fluid/gas dynamics: Navier-Stokes/Euler Equations
- Kinetic modeling of granular flows (with G. Toscani)
- Chemotaxis and formation of biological patterns (with D. Ölz)
- Semiconductor modeling
- Free boundary problems and interfaces
- Reaction-diffusion equations
- Monge-Kantorovich optimal transportation
- Wave equations
- Digital image Processing
- Socio-Economic modeling (with G. Toscanini)

(12)**Nagy, G. 2015. Ordinary Differential Equations. Mathematics Department, Michigan State University, East Lansing, MI, 48824.**
http://users.math.msu.edu/users/gnagy/teaching/ode.pdf Accessed 5 November 2015.

This is a work in progress, but the 331 page first attempt holds forth with theory and good narrative as well as several exercise sets and a reasonable reference section. We quote from the author's Summary. Indeed, the version which is currently up (as of 5 November 2015) was updated 14 September 2015.

> "This is an introduction to ordinary differential equations. We describe the main ideas to solve certain differential equations, like first order scalar equations, second order linear equations, and systems of linear equations. We use power series methods to solve variable coefficients second order linear equations. We introduce Laplace transform methods to find solutions to constant coefficients equations with generalized source functions. We provide a brief introduction to boundary value problems, Sturm-Liouville problems, and Fourier Series expansions. We end these notes solving our first partial differential equation, the Heat Equation. We use the method of separation of variables, hence solutions to the partial differential equation are obtained solving infinitely many ordinary differential equations."

(13)**Tracy, Craig A. 2015. Lectures on Differential Equations.**
https://www.math.ucdavis.edu/~tracy/courses/math22B/math22B.html . Accessed 17 September 2015.

Craig A. Tracy, Distinguished Professor of Mathematics, University of California Davis, has produced a beautiful book with theory and very instructive derivations of physical models with differential equations, both classical and modern. The complete 173 page pdf download can be found at PROF Tracy's course page: https://www.math.ucdavis.edu/~tracy/courses/math22B/math22B.html. Once there click on Lectures on Differential Equations .

This is a set of lecture notes which could serve as a text book in a differential equations course, certainly as a good resource on specific coverages. The text is interlaced with Mathematica and MatLab code for modest command activities. After a topic is introduced almost immediately an application is introduced, e.g., interest accounts after integrating factors. This is followed by conservative system approach to mechanics and Total Energy conservation. The exercise sets are rich applications of the differential equations under study and will pull students further into the use of methods to solve interesting problems.

Difference equations are used to model physical phenomena such as a weighted string to produce the wave equation and the heat equation, while rich and deep detail is offered in quantum harmonic oscillator study.

Along the way many basic fundamental identities are established and used to support theory and furtherance of the applications.

(14) **Chasnov, Jeffrey. 2014. Lecture Notes on Introduction to Differential Equations.** http://www.math.ust.hk/~machas/differential-equations.pdf . Accessed 5 November 2015.

Jeffrey R. Chasnov is a Professor of Mathematics at the Hong Kong University of Science and Technology and his home page http://www.math.ust.hk/~machas/ announces several online texts among them this differential equations text, but also includes Lecture Notes on Mathematical Biology at http://www.math.ust.hk/~machas/mathematical-biology.pdf which contains some very nice mathematics surrounding differential equations issues of population modeling and epidemic modeling. There are no exercises in the mathematical biology book. The notes, Lecture Notes to Numerical Methods, has a chapter on numerical methods in differential equations with MatLab code.

Dr. Chasnov has a rich set of YouTube videos in which he very patiently explains solution strategies and applications. His book Differential Equations with YouTube Examples can be found at http://bookboon.com/en/differential-equations-with-youtube-examples-ebook . Indeed, he has a large presence on YouTube https://www.youtube.com/user/jchasnov/videos in which he offers many solid explanatory videos on techniques and background.

The differential equations text is 128 pages and develops and motivates the theory of the material nicely, with links to YouTube lectures to assist. While there are no exercises this text could serve very well for the fundamentals of differential equations

(15) **Teschl, Gerald. 2012. Ordinaryi Differential Equations and Dynamical Systems.** http://www.mat.univie.ac.at/~gerald/ftp/book-ode/ode.pdf . Accessed 5 November 2015.

The author is a professor of the University of Vienna says in his text," his is a preliminary version of the book

> "Ordinary Differential Equations and Dynamical Systems published by the American Mathematical Society (AMS). This preliminary version is made available with the permission of the AMS and may not be changed, edited, or reposted at any other website without explicit written permission from the author and the AMS."

This 364 page text is a quite theoretical approach and uses Mathematica code to illustrate the study.

16. **Robinson, James. C. An Introduction to Ordinary Differential Equations. Cambridge Universityi Press.** Accessed 5 November 2015. http://faculty.mu.edu.sa/public/uploads/1358974368.6105An_Introduction_to_Ordinary_Differential_Equations.pdf

We quote from the introduction by the author,

> This refreshing, introductory textbook covers standard techniques for solving ordinary differential equations, as well as introducing students to qualitative methods such as phase-plane analysis. The presentation is concise, informal yet rigorous; it can be used for either one-term or one-semester courses.
>
> Topics such as Euler's method, difference equations, the dynamics of the logistic map and the Lorenz equations, demonstrate the vitality of the subject, and provide pointers to further study. The author also encourages a graphical approach to the equations and their solutions, and to that end the book is profusely illustrated. The MATLAB files used to produce many of the figures are provided in an accompanying website.
>
> Numerous worked examples provide motivation for, and illustration of, key ideas and show how to make the transition from theory to practice. Exercises are also provided to test and extend understanding; full solutions for these are available for teachers.

This 415 page work proceeds with a nice mixture of theory and application with lots of graphical illustrations to depict both. There is much attention to the qualitative approach to the study as well.

The solution strategies are offered in the usual order with illustrations from physical models mostly. Although, there are some interesting twists introduced right in the text early in each strategy, e.g.. Newton's Law of Cooling in an unheated building with a varying ambient temperature is studied as an example of integrating factor strategy.

There are digressions to help the reader, e.g., combining two oscillating terms is addressed as a two page aside in a just in time manner.

An entire chapter is devoted to oscillations and the exercises here begin to get rich, e.g., buoyancy problems are offered. An interesting exercise about the undue oscillations in the London Millennium Bride is offered as well. And when it comes to forcing functions there is nice materials e.g., washing machine motion. Resonance is richly discussed with some historical material presented for motivation.

Both series and numerical methods solution strategies are offered in good manner. And there is a nice section on difference equations as well.

The study of linear systems is rich in qualitative analyses, but no real applications. Ecological systems are given a chapter, but again, no application exercises, just a setting for qualitative analyses.

Partial differential equations are not discussed in the text. As such there is no attention to Fourier series either.

The style of the book is almost conversation in places which makes for a good read for students.

17. **Moler, Clive. Ordinary Differential Equaitons. Book Chapter.**
http://www.mathworks.com/moler/odes.pdf . Accessed 5 November 2015.

This is a 53 page Chapter on Ordinary Differential Equations presumably from a larger text byi Clive Moler, the inventor of MatLab, and offers rich illustration of interesting applications using MatLab with code and graphics.

There are some very interesting applications and models here and attention to numerical method when needed is rich.

Bibliographies Containing Differential Equations Resources

(1) Math Forum at Drexel University, 2014. Browse and Search the Library. Home : Math Topics : Differential Equations. Accessed 1 May 2014. http://mathforum.org/library/topics/diffeq/ . List with hot-linked connections to a good number of resources.

(2) Wolfram Library Archiv. 2014. Mathematics-Calculus and Analysis -Differential Equations . Accessed 1 May 2014. http://library.wolfram.com/infocenter/BySubject/Mathematics/CalculusAnalysis/DiffEQ/ . List with hot-linked connections to over 321 resources.

(3) MERLOT (Multimedia Educational resource for Learning and Online Teaching). 2014. Mathematics and Statistics-Mathematics-Differential Equations. Accessed 1 May 2014. http://www.merlot.org/merlot/materials.htm . List with hot-linked connections to a good number of resources.

(4) +Plus Magazine. 2014. Teacher package: Differential equations. Accessed 5 May 2014. http://plus.maths.org/content/teacher-package-differential-equations . Essays on application sof differential equation with good narrative and informed descriptions. No exercises or specific activities.

(5) Mathematics at Texas A&M University. 2014. Resources for Ordinary Differential Equations. Accessed 1 May 2014. http://www.math.tamu.edu/~dallen/ODE_resources.htm . List with hot-linked connections to a good number of resources.

(6) Scheick, Ted. 2014. University of North Carolina Mathematics. Differential Equations, Linear Algebra and Technology. Accessed 1 May 2014. http://www.unc.edu/math/Faculty/scheick/dela.html . List with hot-linked connections to a good number of resources.

(7) SIMIODE. 2014. SIMIODE - Systemic Initiative for Modeling Investigations and Opportunities with Differential Equations. Accessed 1 May 2014. www.simiode.org. An on-line community of teachers and students in which students learn differential equations through modeling and use of technology throughout the learning process. Refereed teaching materials, functioning interest groups, small project groups, project spaces, modeling scenarios, data sets, videos, all devoted to teaching differential equations through modeling and technology.

(8) Reviews - Articles on Differential Equations. 2001. http://www.geneseo.edu/~leary/PCMI2001/. Accessed 6 April 2015. Offers short summaries of recent differential equations articles which have appeared in the *College Mathematics Journal*. Some summaries appeared in *ERIC* or *Math Reviews*.

(9) Hooley, D. E. 2015. http://www.maa.org/publications/periodicals/college-mathematics-journal/the-college-mathematics-journal-index . Accessed 6 April 2015. Under the heading: Differential Equations and Dynamical Systems and further refined as to topics. From the web page,

"All main articles from the *College Mathematics Journal* are listed in the ERIC database, with short abstracts. They may be searched by the author, title, or key words. Classroom capsules, notes, software reviews, and FFF items are not in ERIC at Columbus. "

(10) Mathews, J. 2015. http://mathfaculty.fullerton.edu/mathews/n2003/NumericalUndergradRes.html . Accessed 5 April 2015. From Numerical Analysis - Numerical Methods prepared for Research Experience for Undergraduates we find lots of reference d materials w.r.t. numerical methods and theory in differential equations.

(11)Zhiao, Di. 2010. Differential Equation Models for Systems Biology: A Survey. Contains over 92 references in the area of differential equations and systems biology and offers rich illustrations of same in the 30 pages of this document. http://www.advancedcomputing.cn/sys_bio_review.pdf. Accessed 15 May 2015.

Videos

(1) Gregg Waterman YouTube Channel. Accessed 5 March 2015.
https://www.youtube.com/user/watermanOIT

Very careful, considerate, and motivated presentations. Could be used to introduce or review materials.

There are many worked examples and lessons. We list them here by name and number: Laplace Transform (16), Linear Algebra (13), Ordinary Differential Equations (12), and Systems of Differential Equations (4).

(2) Khan Academy Accessed 8 March 2015.
https://www.khanacademy.org/

Here one finds informal, yet technically competent, descriptions of solving strategies presented. Sometimes the chat slows things down, but students might like the chat. The coverage is great in terms of topics.

(3) Mathispower4u (James Sousa). Accessed 8 March 2015.
http://mathispower4u.yolasite.com/diff-eq.php .

Once at the above website you get to a full, and I mean full, panorama of YouTube lessons which are terrifically good. Exceptionally clear and complete set of videos on just about all DE topics. Very well presented with clear examples and reasoned steps for students to follow. Wonderful stuff.

(4) Patrick JMT -Just Math Tutoring (Patrick Jones). Accessed 8 March 2015.
http://patrickjmt.com/

Videos of worked examples with explanation are offered for differential equations techniques. Presentations are good, not great, and do offer some intuitive ideas as well as algorithmic approaches.

Video sites coming from other fields

(1) Flipping Physics. Accessed 8 March 2015.
https://www.youtube.com/user/flippingphysics

Very clever videos made by one high school teacher who plays many parts. There are a number of experiments, good many physics/mathematics concepts discusses, lots of chatter which is quite engaging and humorous, and high quality video.

(2) Physics with Mr. Scott Secrest. Accessed 8 March 2015.
https://www.youtube.com/channel/UCRGLooRNQWu9Fo4KhBlSEIA

Combination of demonstrations, derivations, solutions, and good patient explanations with plausibility approach and follow on "What does solution imply?"

(3) Video Lectures in Mathematics
https://www.pinterest.com/mathematicsprof/

This site contains links to math videos. The PORTAL boards (last boards) link to sites containing not one, but several videos. You can post your own videos also. One needs to register. Many items are pointers to, say, YouTube, for example Nonlinear Dynamics and Chaos - Steven Strogatz, Cornell University pointed to at https://www.youtube.com/playlist?list=PLbN57C5Zdl6j_qJA-pARJnKsmROzPnO9V where we find, "This course of 25 lectures, filmed at Cornell University in Spring 2014, is intended for newcomers to nonlinear dynamics and chaos. It closely follows Prof. Strogatz's book, `Nonlinear Dynamics and Chaos: With Applications to Physics..'"

Sources Containing Modeling Resources

(1) Gross, Louis and Michael Knorrenschild. 2013. Text Books on Mathematical Modeling in Biology. Compiled from the Internet by Michael Knorrenschild. Modified by Louis Gross, Oct. 1995, May 2000, March 2001, June 2003. Accessed 1 May 2014. http://www.tiem.utk.edu/~gross/math.modeling.books.txt.fmt . Thoroughly annotated list of materials as of 2003. May be updated somewhere, though.

(2) CODEE – Community of Ordinary Differential Equations Educators. 2015. http://www.codee.org/ . Accessed 8 March 2015.
This NSF supported activity offers (a) Digital Library - an online repository of materials related to the teaching and learning of ordinary differential equations, (b) the CODDE Journal for publications of materials, and (c) a newsletter. While not currently actively growing the site has interesting materials.

(3) COMAP. 2015. www.comap.com. Accessed 5 April 2015.
This community has been offering modeling materials for years. Some materials are free, but there is a yearly subscription fee which will give the user access the *The UMAP Journal* (in which modeling and teaching mathematics articles appear often as do editorials and discussions about undergraduate teaching of mathematics); mathematics modeling materials from world-wide modeling competitions; all UMAP Modules (and these are great sources of actual classroom useable activities); and other materials related to modeling both in undergraduate efforts and high school environment.

(4) Murray, J. D. 2002. *Mathematical Biology: I. An Introduction, Third Edition*. Springer. http://www.ift.unesp.br/users/mmenezes/mathbio.pdf . Accessed 14 May 2015. While this classic book is still on the commercial market there exists this complete on-line version freely available. There are many population models with lots of variety, management, delay, logistic, limit cycle, predator-prey, competitive exclusion, marital interactions, kinetics, oscillators and switches, infectious diseases, diffusion, biological waves, and fractals in this 576 page tome.

(5) Ryu, Jaeku. 2015. ShareTechNote. Engineering Math Math – Differential Equations. http://www.sharetechnote.com/html/EngMath_DifferentialEquation.html . Accessed on 31 May 2015.

The author and maintainer of this wonderful site is a Protocol Software Engineer at Anritsu, Ottawa, Canada Area specializing in telecommunications.

This is a remarkable set of rich resources for engineering education and we note the strong conviction of the author(s) as to the role of differential equations in engineering, for from the introduction we quote:

> "I would say to many of friends and juniors working in engineering area. `If you are good at two topics, Matrix and Differential Equations, you would be good at ANY of the engineering field.' Of course, just being good at math would not

> automatically lead you to success in the field, but we cannot deny the fact that mathematical understanding is very crucial in most of engineering.
>
> "In this page, I would like to explain the basic concept of differential equation and it's applications as easy as I can do, but in some cases it is hard to avoid tackling the mathematical equations directly."

There are many engineering specialties offered at this site and all re rich in illustrations, example, narrative in support of theory and practice. However, what is best is the flow of the narrative and the rich illustrations and step-by-step. Example after example are offered from many fields of engineering and pharmacokinetics as well with Governing Law principle first and then a gentle building of the differential equation with examples to show the model in action.

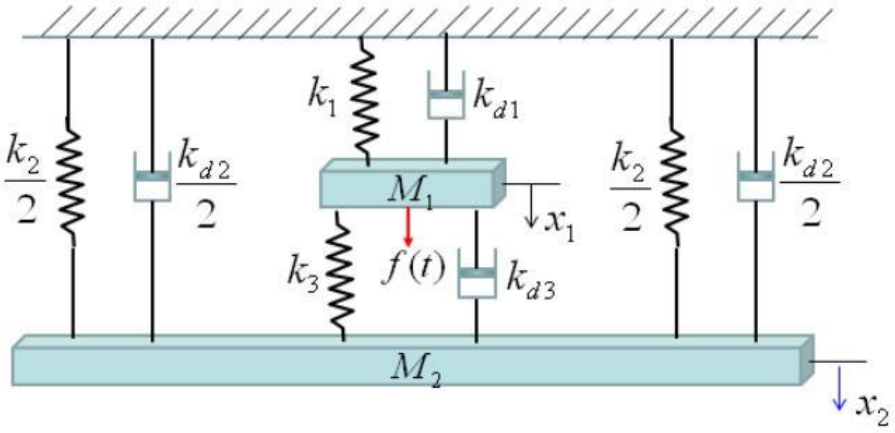
The site is also rich with a very remarkable Personal Interest page in which the author(s) give a list of questions and resources as well as favorites that will fascinate and drawn in the reader.

> "Following are some of the questions I spent most of my time, contemplating, reading, watching when I am not working on engineering/techy stuffs. These are more for myself not to forget about the thread of my thought. Just in case there is somebody who is seeking to have the answer to the same questions… or have interests in the similar topics in this page, just try to use these as a kind of keywords to search more information in your search engine. I am also using this page as a keyword finder for further search. Now as I get older, I found myself more absent-minded. My brain requires more frequent 'refresh' or sometimes 'rebooting' :)"

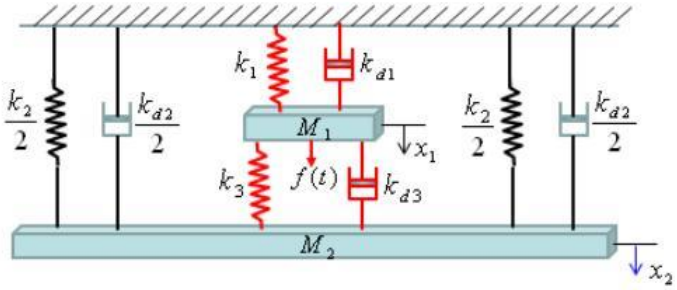
We illustrate (with a screen capture) the flare offered in this one example (or the first part of this one example) on a Vehicle Suspension System in which the differential equation model is built carefully and meticulously.

< Example : Coupled spring - Vehicle Suspension System >

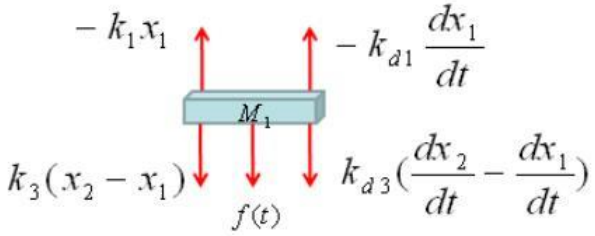
You can practice what you learned from the previous two examples and this is the one that can be easily extended for a real life problem. You can easily apply this example to model a suspension system of a vehicle.
It may look a little bit scary, but the logic of the modeling is always the same however complex system it is.
Do you remember the logic (process) ?
i) Break down the system into each component. (When you see this kind of spring-mass system, each Mass is the building block of the system).
ii) Draw the arrows (vectors) to represent the direction of Forces being applied to each component.
iii) Write down mathematical formula for each of the arrows (vectors).
iv) Combine all the component formula into a single differential equation



Now Let's start with the first component. Can you identify the component ? M1 is the first component. Mark all the springs, damper and applied force for the component as shown below.



Now draw arrows (vectors) to represent forces being aplied to the component (Mass) as shown below.



Now combine each component formula into single differential equation as shown below.

$$M_1\frac{d^2x_1}{dt^2} = -k_1x_1 - k_{d1}\frac{dx_1}{dt} + k_3(x_2 - x_1) + k_{d3}(\frac{dx_2}{dt} - \frac{dx_1}{dt}) + f(t)$$

(6) LeMasurier, Michelle. 2006. Nonstandard Topics for Student Presentations in Differential Equations. *PRIMUS*. 16(4): 349-357.

This is an amazing paper in which the author offers 33 presentation topics with full paginated citations from 27 references. At Hamilton College, where the author teaches, students give oral presentations and this material provides a wealth of options for them, but in the context of SIMIODE and modeling first differential equations we are given a real treasure chest of rich resources. No doubt your library has access to this well-established and rich journal, *PRIMUS - Problems, Resource, and Issues in Mathematics Undergraduate Studies*. Get this article!!!

We quote the Abstract of the paper, "An interesting and effective way to showcase the wide variety of fields to which differential equations can be applied is to have students give short oral presentations on a specific application. These talks, which have been presented by 30–40 students per year in our differential equations classes, provide exposure to a diverse array of topics that are generally not covered in a typical differential equations course, and include applications in biology, economics, chemistry, ecology and physics. We will discuss these presentations and provide a list of (classroom tested) presentation topics along with their references, which are from textbooks readily available in most school libraries."

Quick, Short, Direct Modeling Descriptions

(1) British Columbia Institute of Technology, Math Department. 2014. Tables of Examples of Applications to Various Areas of Technology. Contains five short illustrations. Accessed 1 May 2014. http://commons.bcit.ca/math/examples/ .

Class Notes

(1) Dawkins, Paul. 2014. Paul's Online Math Notes - Differential Equations . Accessed 1 May 2014. http://tutorial.math.lamar.edu/Classes/DE/DE.aspx.
Engaging style of presenting traditional materials with a few, limited applications in such areas as vibrations.

(2) Grigg, Nathan. 2015. Differential Equation Lessons. From Nathan Grigg's Teaching Archives http://archive.nathangrigg.net/teaching/diff-eq/lessons/ . Accessed 5 May 2015.
Several topics, standards like separation of variables, integrating factors, Euler's method, Laplace Transform. Also some modeling problems, like tank with dyed water and credit card interest, are offered in lecture note with videos of problems and models worked.

(3) Olds, Victoria. 2000. Unit 10 – First Order Differential Equations with Applications. http://www-home.math.uwo.ca/~volds/math030notes/Unit10.pdf . Accessed 14 May 2015.
These are some straightforward narrative notes offering solution techniques with explanation and illustration along with examples worked out and traditional illustrations of elementary modeling, e.g., carbon dating, cooling, mixing, and falling object.

(4) Mathematical Models. 2013. Section 1.1-1.3 Mathematical Models Notes. http://www.math.ucsd.edu/~pkolesni/math20d/math20D_1.2.pdf . Accessed 14 May 2015.
This is a very nice set of overheads to illustrate the steps in modeling from naturual laws with differential equations.

(5) Panfilov, Alexander. 2010. Qualitative Analysis of Differential Equations. Theoretical Biology, Utrecht University, Utrecht Netherlands.
http://www-binf.bio.uu.nl/panfilov/bioinformatica/bioinf10.pdf . Accessed 15 May 2015.
Notes and exercises for seven day course with complete introduction to mathematical essentials, definitions, illustrations, examples, exercises, dictionary, and rich illustrations of qualitaltive study approach.

(6) 2010. Biomedical Engineering Course Notes. BME2305. Chapter 7 of CDM, *Numerical Modeling in Biomedical Systems* – Dynamic Systems of Ordinary Differential Equations. http://coewww.rutgers.edu/classes/bme/bme305/ . Accessed 15 May 2015. Offers motivated (by biomedical examples) illustrations of numerical concepts in solutions of ordinary differential equations. Covers broad dynamical systems approach with examples as well. Offers MatLab code. A set of problems are firmly rooted in biological applications and require numerical efforts.

(7) Mahaffy, Joseph M. 2013. Calculus for the Life Sciences – Lectures Noes – Introduction to Differential Equations. San Diego State University. http://www-rohan.sdsu.edu/~jmahaffy/courses/f11/math122/beamer_lectures/intro_de.pdf Accessed 15 May 2015. Contains lots of illustrations and definitions and presents models from Malthusian Growth, Springs, Evaporation. 128 frames carefully organized.

Good Questions for Students

1. MathQuest. 2013. Resources for Clickers and Classroom Voting in Collegiate Mathematics. Carroll College, Helena MT USA. http://mathquest.carroll.edu/ . Accessed 4 May 2015.

   This is an exceptional list of meaningful and thoughtful questions on which students can vote (i.e. multiple choice) in many areas of mathematics and we specifically single out the hundreds of questions in the area of differential equations.

   From the web page,

   > "This page contains resources for classroom voting in mathematics, also known as ConcepTests for Peer Instruction or "Clicker Questions". These materials were developed through two NSF-funded projects: "MathQUEST: Math Questions to Engage Students" (2006-2009) and "MathVote: Teaching Mathematics with Classroom Voting" (2010-2013). These projects involve not only creating and testing libraries of classroom voting questions, but also involve the study of these teaching methods. For more details see our Project Summaries below. These projects have been run here at Carroll College's, Department of Mathematics, Engineering, and Computer Science by Holly Zullo and Kelly Cline.

   Under Question Libraries for Specific Courses we find hundreds of thoughtful questions (all multiple choice) in which students have to really think to answer. Moreover, there is an opportunity to obtain teacher's editions of the questions in which discussion about the issues raised or the answers or incorrect answers are offered.

   Again, we quote from the web page

   > Classroom Voting Questions: Differential Equations
   >
   > Please e-mail us if you intend to use any of these questions! kcline@carroll.edu. We will send you a teacher's edition and we would really like your feedback!
   >
   > Click Here for the complete set of voting questions for differential equations.
   >
   > Click Here for these questions in a larger font, which may be more useful if you plan to cut and paste these questions from the pdf file into PowerPoint.
   >
   > Click Here, for the differential equations and linear algebra voting questions ordered following Farlow, Hall, McDill, & West's "Differential Equations and Linear Algebra" 2nd edition.

Questions by topic:

- What is a Differential Equation?, **Sample Teacher's Edition**
- Slope Fields and Euler's Method
- Separation of Variables
- Exponential Solutions, Growth and Decay
- Equlibria and Stability **Sample Teacher's Edition**
- First Order Linear Models
- Logistic Models
- Nonhomogeneous Differential Equations & Undetermined Coefficients
- Second Order Differential Equations: Oscillations
- Mixing Models
- Existence & Uniqueness
- Bifurcations **Sample Teacher's Edition**
- Linear Operators
- Second Order Differential Equations: Damping
- Linear Combinations and Independence of Functions
- Second Order Differential Equations: Forcing **Sample Teacher's Edition**
- Beats and Resonance
- 2nd Order Differential Equations as Systems
- Phase Portraits and Vector Fields
- Testing Solutions to Linear Systems
- Euler's Method and Systems of Equations
- Modeling with Systems **Sample Teacher's Edition**
- Solutions to Linear Systems
- Geometry of Systems
- Nonhomogeneous Linear Systems
- Nonlinear Systems
- Introduction to Partial Differential Equations
- Integrating Factors
- Power Series Solutions
- Laplace Transforms

Again, we quote from the web page:

> These are the libraries that we use here at Carroll College. For libraries of classroom voting resources in mathematics from people at other institutions see our Resources page, which has links to libraries for Statistics, College Algebra, Group Theory, and more.

2. NO Source Attributed. 2011. Chapter 4. Applications of Differential Equations. https://lupucezar.files.wordpress.com/2011/02/attachment-1.pdf . Accessed 14 May 2015. Contains some short models worked out and a set of textbook excercises byi discipline, e.g., chemistry, investment, medicine.

Presentations

(1) Shampine, L. F. Date Not Provided. Delay Differential Equations. http://faculty.smu.edu/shampine/Read1.pdf . Accessed 15 May 2015.
Slides from presentation at Southern Methodist University. Presents numerical methods – practice, theory, and MatLab code.

(2) Lie, Knut-Andreas. 2005. Introduction to Mathematical Modelling: Ordinary Differential Equations and Heat Equations. http://www.uio.no/studier/emner/matnat/ifi/INF2340/v05/foiler/sim02.pdf . Accessed 18 May 2015.
Slides for materials on basic ordinary differential equations and development of heat equation as well as intensive material on numerical methods for solving heat equation.

Software

R

r-project. 2015. Accessed 8 March 2015.
http://www.r-project.org/ .

(1) Soetaert, Karline , Thomas Petzoldt, and R. Woodrow Setzer. 2010. Solving Differential Equations in R. Accessed 1 Mary 2014. *The R Journal.* 2(2): 5-15.

Abstract: Although R is still predominantly applied for statistical analysis and graphical representation, it is rapidly becoming more suitable for mathematical computing. One of the fields where considerable progress has been made recently is the solution of differential equations. Here we give a brief overview of differential equations that can now be solved by R.

(2) Ellner, S. O. and J. Guckenheimer. 2011. An introduction to R for dynamic models in biology. https://people.cam.cornell.edu/~dmb/DynamicModelsLabsInR.pdf . Accessed 15 May 2015. Offers code and basic narrative about solving differential equations systems, phase plane analysis, Markov simulations, general simulations, The authors say, "These notes for computer labs accompany our textbook *Dynamic Models in Biology* (Princeton University Press 2006), but they can also be used as a standalone introduction to R as a scripting language for simulating dynamic models of biological systems."

Maple

MapleSoft. 2014. User Community-Application Center-Mathematics- Differential Equation. Accessed 1 May 2014. http://www.maplesoft.com/applications/category.aspx?CID=136 . Some 76 differential equations applications using Maple computer algebra system.

Mathematica

Wolfram Research. 2014. Wolfram Alpha Topics – Differential Equations. Accessed 5 May 2014. http://www.wolframalpha.com/examples/DifferentialEquations.html. Listings of commands for Wolfram Alpha related to differential equations.

Wolfram Research. 2014. Differential equations demonstrations. http://demonstrations.wolfram.com/topic.html?topic=Differential+Equations . Over 320 applications with complete code for Mathematica applications in differential equations.

Matlab

Matlab. 2015. Accessed 8 March 2015.
http://www.mathworks.com/ .
From the web page "MATLAB and Simulink are the tools of inspiration and innovation, used at more than 5000 universities worldwide." There are examples, courseware, and videos.

Maxima

Maxima, a Computer Algebra Syilstem. Accessed 8 March 2015.
http://maxima.sourceforge.net/ .

FREE software package based on MACSYMA. From the web page, “Maxima is a system for the manipulation of symbolic and numerical expressions, including differentiation, integration, Taylor series, Laplace transforms, ordinary differential equations, systems of linear equations, polynomials, sets, lists, vectors, matrices and tensors.

“Maxima yields high precision numerical results by using exact fractions, arbitrary-precision integers and variable-precision floating-point numbers. Maxima can plot functions and data in two and three dimensions.”

MLAB

Civilized Software, Inc. Accessed 16 September 2015**.**
www.civilized.com

MLAB, an advanced mathematical and statistical modeling system, is an ideal tool for mathematical and statistical exploration, and for solving simulation and modeling problems such as chemical kinetics, pharmacological compartmental models, multiple site ligand binding, neurophysiological modeling, and ultracentrifuge models, to name just a few. MLAB is especially designed to handle differential equation models.

There are versions of MLAB for PC (DOS, Windows3.1, Windows95/98/NT/2000/XP/Vista/Win7, Linux), Macintosh (PPC OS 8.x, OS 9, OS X-classic mode (emulation)), and OS X-native mode for both Intel-Macs and PPC-Macs, older Motorola Macintosh systems, SGI, and NeXT (Motorola, Intel, and HP) platforms.

Descriptive material plus a collection of technical reports showing the application of MLAB are available. Please take a look at whatever you wish from the selections offered. Especially note the Technical Application Notes and Examples!

The rich set of materials and mathematical support papers are worth attention to anyone teaching a differential equations course or modeling effort. The background and theory are offered as are algorithms for parameter estimation and data analysis. We publish two very good support papers from the developers on line at www.simiode.org .

"Chemical Kinetics - Simple Binding," https://www.simiode.org/resources/1222

"Numerical to Closed Form Solution - Derivation," https://www.simiode.org/resources/1220 .

Octave

Gnu Octave. 2015. Accessed 8 March 105.
https://www.gnu.org/software/octave/ .

From the web site, “GNU Octave is a high-level interpreted language, primarily intended for numerical computations. It provides capabilities for the numerical solution of linear and nonlinear problems, and for performing other numerical experiments. It also provides extensive graphics capabilities for data visualization and manipulation. Octave is normally used through its interactive

command line interface, but it can also be used to write non-interactive programs. The Octave language is quite similar to Matlab so that most programs are easily portable.

"Octave is distributed under the terms of the GNU General Public License."

SAGE

SAGE. 2015. Accessed 8 March 2015.
http://www.sagemath.org/ .
FREE software package with growing community of users. From the web page, "SageMath **is** a free open-source mathematics software system licensed under the GPL. It builds on top of many existing open-source packages: NumPy, SciPy, matplotlib, Sympy, Maxima, GAP, FLINT, R and many more. Access their combined power through a common, Python-based language or directly via interfaces or wrappers. Mission: Creating a viable free open source alternative to Magma, Maple, Mathematica and Matlab.

(1) SAGE. Undated. Direction Fields of differential equations...with SAGE . http://www.ms.uky.edu/~ma138/Spring15/Sage_instruction.pd . Accessed 15 Mary 2015. Offers thorough guid as to how to study direction fields with SAGE.

On Line Courses and Notes

(1) MIT Open Courseware. 2014. (Ordinary) Differential Equations. Taught by Arthur Mattuck. Accessed 1 May 2014. http://ocw.mit.edu/courses/mathematics/18-03-differential-equations-spring-2010/. Complete course resources including videos of all lectures, course notes, Mathlets, Exams, Recitation Section Material, Solutions.

(2) S.O.S. MATHematics. 2014. Differential equations. Accessed 1 May 2014. http://www.sosmath.com/diffeq/diffeq.html . Short and sweet coverage of major topics, but with no exercises and few models.

(3) Veeh, Jerry Alan. 2002. Lecture Notes on Ordinary Differential Equations. Accessed 5 May 2014. http://language6.com/l/lecture-notes-on-ordinary-differential-equations-w3299.html . An 88 page set of notes covering basics through Laplace transforms with examples and exercises.

(4) Zeilberter, Doron. 2014. Differential Equations for Engineering and Physics. Accessed 1 May 2014. http://www.math.rutgers.edu/~zeilberg/math244_13.html . Complete lecture notes with exams, quizzes, and solutions.

(5) TeachersPayTeachers. 2015. https://www.teacherspayteachers.com/ . Accessed 5 April 2015. This site (for which you need to register – but free) has teaching materials from all levels including differential equations. Idea is for teachers to post materials for modest sale price, e.g., $1.00 - $16.00, that other teachers might use. Material ranges from Jeopardy game reviews, to lessons, to problems.

(6) The PostCALC Project. 2015. https://www.math.duke.edu//education/postcalc/index.html. Accessed 5 April 2015. This site offers, "Interactive modules for high school students who have finished calculus" and included modules on SIR model for Spread of Disease, Models of Specie Interaction, and Population Growth Models.

(7) OpenLearn. 2015. The home of free learning from The Open University. http://www.open.edu/openlearn/science-maths-technology/mathematics-and-statistics/mathematics-education . Accessed 6 May 2015. This site offers some portions Free OpenLearn courses of Open University in modeling and differential equations. Such units as *Modelling pollution in the Great Lakes*, *Analysing skid marks* and *Developing modelling skills* are given freely and have worthwhile reads and examples as well as some exercises with solutions. A unit called, "Modelling with first order differential equations," offers a workbook on problems involving motion of a particle in one dimension, friction, air and water resistance. Detailed narrative and structure development make this a nice student read.

(8) Blomhøj, M., T.H. Kjeldsen, and J. Ottesen. 2014. Compartment Models. http://www4.ncsu.edu/~msolufse/Compartmentmodels.pdf . Accessed 15 May 2015. This is a Chapter on compartment models from unknown text. From first page we quote the authors, "Background: It is important to master the ability to develop models. Modeling of dynamical

systems plays a very important role in applied science, and compartment models are among the most important tools used for analyzing dynamical systems.
Aim: The aim with this chapter is to learn more about development of compartment models."

(9) Vas, Lia. No date. System of Differential Equations. http://www.usciences.edu/~lvas/Math320/Systems.pdf . Accessed 15 May 2015. This 19 page notes set offered instruction on how to solve and plot Systems of equations with emphasis on phase plane analysis in MatLab.

(10) Sasane, Amol. No Date. http://personal.lse.ac.uk/sasane/ma209.pdf . Accessed 15 May 2015. This 129 page set of notes is about broad general theory with no applications in which Maple is the presenting software. Emphasis is on stability and qualitative behaviors.

Internet Blogs, Essays, Thoughts, and General Resources

(1) CODEE: Community of Differential Equations Educators. 2014. CODEE Digital Library. Accessed 6 May 2014. http://www.codee.org/. Online digital *CODEE Journal* offering resources and discussions on use of materials for teaching differential equations.

(2) College Math Teaching: Differential Equations. 2014. Accessed 5 May 2014. http://collegemathteaching.wordpress.com/category/differential-equations/ . Over 26 Blog entries on teaching differential equation in which many issues are raised.

(3) Biegert, Mark. and Math EncountersBlog. 2015. http://mathscinotes.com/ . Accessed 6 April 2015. The blog author says, **"**I am an electrical engineer by training who is currently working as a hardware development director for a telecommunications company. Over the years, I have become more and more impressed with the ability of relatively small amounts of mathematics to help shed light on significant technical problems.

"There are many types of mathematical analyses. They range from very detailed modeling exercises that are impressive, but are really for specialists, to "Fermi problems" that are exercises in gross approximation. This blog will steer a middle course and will look at the simple mathematics that crops up in the daily life of a working engineer."

Five very nice blog entries, with lots of materials, are offered when one searches with "differential equation."

(4) Ingalls, Brian . 2012. Mathematical Modelling in Systems Biology: An Introduction. http://www.math.uwaterloo.ca/~bingalls/MMSB/Notes.pdf . Accessed 13 May 2015. This is a broad set of notes with rich illustration of differential equations applied in systems biology including compartment models, chemical reactions, metabolism, and gene regulations. The work contains some 386 pages of rich illustrations.

Teaching and Learning Differential Equations

(1) Rasmussen, C. and K. Whitehead. 2015. Research Sampler 7: Learning and Teaching Ordinary Differential Equations. http://www.maa.org/programs/faculty-and-departments/curriculum-department-guidelines-recommendations/teaching-and-learning/learning-and-teaching-ordinary-differential-equations Accessed 5 April 2015. Sections covered are: Making Connections between Representations, What Lies Beneath Correct Answers?, Numerical Approximations and Graphical Prediction, Proofs of Graphical Predictions, The Cultural Status of Graphical Representations, Student Understanding of Systems and Second Order Differential Equations, Technology, The Differential Equations Classroom, To Sum Up, and References.

(2) Wangler, T. G. 2015. Paradigm Lost: A Modern Approach to Teaching Ordinary Differential Equations. http://archives.math.utk.edu/ICTCM/VOL08/C059/paper.pdf . Acessed 5 June 2015.Thoughtful presentation about the general milieu of teaching differential equations.

(3) Nelson, D. 2015. Differential Equations and a Poster Project. http://abacus.bates.edu/~dnelson/DE_port.pdf . Accessed 5 June 2015. This is a self-contained handout for students at Bates College in which Professor Nelson offers complete and very specific instructions on a Poster Project activity in differential equations. Included is a nice set of topics and a complete bibliography in support of these projects.