

Table of Laplace Transforms

Powers

| $f(t)$ | $F(s) = \mathcal{L}\{f\}(s)$ | $f(t)$ | $F(s) = \mathcal{L}\{f\}(s)$ |
|--------|--|------------|--|
| 1 | $\frac{1}{s}$ | $t^{-1/2}$ | $\sqrt{\frac{\pi}{s}}$ |
| t | $\frac{1}{s^2}$ | $t^{1/2}$ | $\frac{\sqrt{\pi}}{2s^{3/2}}$ |
| t^n | $\frac{n!}{s^{n+1}}, n$ a positive integer | t^α | $\frac{\Gamma(\alpha + 1)}{s^{\alpha+1}}, \alpha > -1$ |

Trigonometric Function

| $f(t)$ | $F(s) = \mathcal{L}\{f\}(s)$ |
|----------------------------|------------------------------------|
| $\sin kt$ | $\frac{k}{s^2 + k^2}$ |
| $\cos kt$ | $\frac{s}{s^2 + k^2}$ |
| $\sin^2 kt$ | $\frac{2k^2}{s(s^2 + 4k^2)}$ |
| $\cos^2 kt$ | $\frac{s^2 + 2k^2}{s(s^2 + 4k^2)}$ |
| $t \sin kt$ | $\frac{2ks}{(s^2 + k^2)^2}$ |
| $t \cos kt$ | $\frac{s^2 - k^2}{(s^2 + k^2)^2}$ |
| $\frac{2(1 - \cos kt)}{t}$ | $\ln \frac{s^2 + k^2}{s^2}$ |
| $\frac{\sin at}{t}$ | $\arctan\left(\frac{a}{s}\right)$ |

Trigonometric Function

| $f(t)$ | $F(s) = \mathcal{L}\{f\}(s)$ |
|---|---|
| $\sin kt + kt \cos kt$ | $\frac{2ks^2}{(s^2 + k^2)^2}$ |
| $\sin kt - kt \cos kt$ | $\frac{2k^3}{(s^2 - k^2)^2}$ |
| $1 - \cos kt$ | $\frac{k^2}{s(s^2 + k^2)}$ |
| $kt - \sin kt$ | $\frac{k^3}{s^2(s^2 + k^2)}$ |
| $\frac{a \sin bt - b \sin at}{ab(a^2 - b^2)}$ | $\frac{1}{(s^2 + a^2)(s^2 + b^2)}$ |
| $\frac{\cos bt - \cos at}{a^2 - b^2}$ | $\frac{s}{(s^2 + a^2)(s^2 + b^2)}$ |
| $\frac{\sin at \cos bt}{t}$ | $\frac{1}{2} \arctan \frac{a+b}{s} + \frac{1}{2} \arctan \frac{a-b}{s}$ |

Hyperbolic Functions

| $f(t)$ | $F(s) = \mathcal{L}\{f\}(s)$ |
|--------------|------------------------------------|
| $\sinh kt$ | $\frac{k}{s^2 - k^2}$ |
| $\cosh kt$ | $\frac{s}{s^2 - k^2}$ |
| $\sinh^2 kt$ | $\frac{2k^2}{s(s^2 - 4k^2)}$ |
| $\cosh^2 kt$ | $\frac{s^2 - 2k^2}{s(s^2 - 4k^2)}$ |

| $f(t)$ | $F(s) = \mathcal{L}\{f\}(s)$ |
|-----------------------------|-----------------------------------|
| $t \sinh kt$ | $\frac{2ks}{(s^2 - k^2)^2}$ |
| $t \cosh kt$ | $\frac{s^2 + k^2}{(s^2 - k^2)^2}$ |
| $\frac{2(1 - \cosh kt)}{t}$ | $\ln \frac{s^2 - k^2}{s^2}$ |
| | |

Exponential Functions

| $f(t)$ | $F(s) = \mathcal{L}\{f\}(s)$ |
|-----------------------------|---|
| e^{at} | $\frac{1}{s-a}$ |
| te^{at} | $\frac{1}{(s-a)^2}$ |
| $t^n e^{at}$ | $\frac{n!}{(s-a)^{n+1}}$, n a positive integer |
| $\frac{e^{bt} - e^{at}}{t}$ | $\ln \frac{s-a}{s-b}$ |

| $f(t)$ | $F(s) = \mathcal{L}\{f\}(s)$ |
|---|-----------------------------------|
| $\frac{1}{\sqrt{\pi t}} e^{-a^2/4t}$ | $\frac{e^{-a\sqrt{s}}}{\sqrt{s}}$ |
| $\frac{a}{2\sqrt{\pi t^3}} e^{-a^2/4t}$ | $e^{-a\sqrt{s}}$ |
| $\frac{e^{at} - e^{bt}}{a-b}$ | $\frac{1}{(s-a)(s-b)}$ |
| $\frac{ae^{at} - be^{bt}}{a-b}$ | $\frac{s}{(s-a)(s-b)}$ |

Exponential and Trigonometric Functions

| $f(t)$ | $F(s) = \mathcal{L}\{f\}(s)$ |
|------------------|------------------------------|
| $e^{at} \sin kt$ | $\frac{k}{(s-a)^2 + k^2}$ |
| $e^{at} \cos kt$ | $\frac{s-a}{(s-a)^2 + k^2}$ |

Exponential and Hyperbolic Functions

| $f(t)$ | $F(s) = \mathcal{L}\{f\}(s)$ |
|-------------------|------------------------------|
| $e^{at} \sinh kt$ | $\frac{k}{(s-a)^2 - k^2}$ |
| $e^{at} \cosh kt$ | $\frac{s-a}{(s-a)^2 - k^2}$ |

Trigonometric and Hyperbolic Functions

| $f(t)$ | $F(s) = \mathcal{L}\{f\}(s)$ |
|--------------------|------------------------------------|
| $\sin kt \sinh kt$ | $\frac{2k^2 s}{s^2 + 4k^4}$ |
| $\sin kt \cosh kt$ | $\frac{k(s^2 + 2k^2)}{s^4 + 4k^4}$ |
| $\cos kt \sinh kt$ | $\frac{k(s^2 - 2k^2)}{s^4 + 4k^4}$ |
| $\cos kt \cosh kt$ | $\frac{s^3}{s^4 + 4k^4}$ |

Unit Step Function $H(t)$

| $f(t)$ | $F(s) = \mathcal{L}\{f\}(s)$ |
|----------------|------------------------------|
| $f(t-a)H(t-a)$ | $e^{-as}F(s)$ |
| $H(t-a)$ | $\frac{e^{-as}}{s}$ |

General Functions

| | |
|------------------------------------|---|
| $f(t)$ | $F(s) = \mathcal{L}\{f\}(s)$ |
| $e^{at}f(t)$ | $F(s - a)$ |
| $f(t - a)H(t - a)$ | $e^{-as}F(s)$ |
| $f^{(n)}(t)$ | $s^n F(s) - s^{(n-1)}f(0) - \dots - f^{(n-1)}(0)$ |
| $t^n f(t)$ | $(-1)^n \frac{d^n}{ds^n} F(s)$ |
| $\int_0^t f(\tau)g(t - \tau)d\tau$ | $F(s)G(s)$ |